CHAPTER 20
MECHATRONICS

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Mechatronics is the integration of computers, electronics, and information sciences into mechanical engineering. The advent of inexpensive and small microcomputers and electronics has led to the widespread integration into mechanical systems. The goal of this integration is to improve the performance of mechanical systems. Today's mechanical engineers cannot design state-of-the art systems without considering the cross-disciplinary field of mechatronics.

A typical mechatronic system is depicted in Fig. 1. It shows a mechanical system whose performance is monitored by some type of sensor. The sensor output is conditioned so it can be input into a microcomputer usually embedded in the product. The microcomputer determines an input to the mechanical system that will improve performance. The microcomputer outputs this information, and this output is converted and amplified to cause an actuator to alter the behavior of the mechanical system. This sense–plan–act cycle is repeated (usually several times per second) to improve the performance of the mechanical system.

Knowledge of mechatronics first and foremost requires a solid understanding of mechanical engineering. Mechanical engineering and control systems are covered throughout this book so this chapter will give only a basic overview of electrical engineering and computer engineering.

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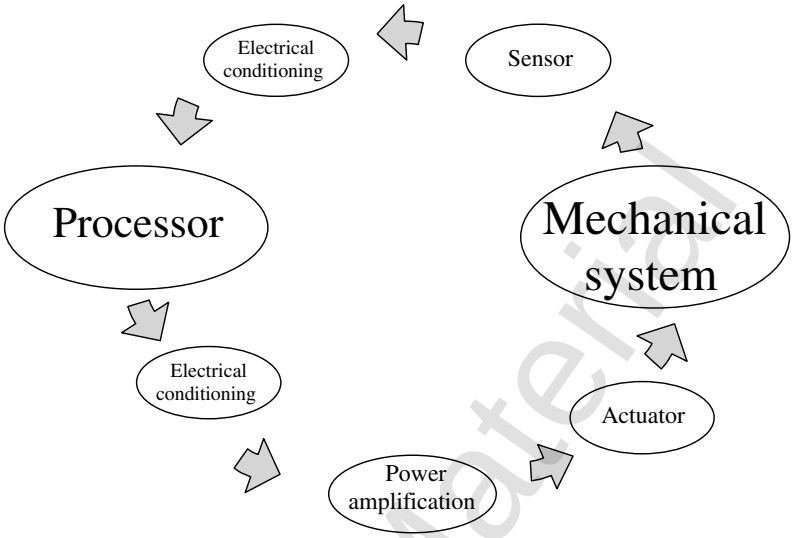


Figure 1 Mechatronic system.

1 BASIC ANALOG ELECTRONICS

Basic circuit analysis and design are critical in the field of mechatronics. With improving computer technology it is tempting to treat all systems as digital; however, this is an analog world. Analog circuits are often the best choice to condition and modify signals to and from analog actuators and sensors.

1.1 Definitions

Engineering modeling and analysis are often described as the analysis of the flow of power between various elements. The state of an engineering element can be defined by two variables—the effort variable and the flow variable. The instantaneous power for that element is defined as the product of the effort variable and the flow variable. For example, in traditional linear mechanical systems the power is defined as the product of the velocity of a mass and the net applied force to that mass. In fluid systems this can be written as the product of the volumetric flow rate and the pressure.

These analogies are often helpful to mechanical engineers trying to understand electrical circuits. In electrical systems the power is defined as the product of the electrical current (amperage) and the electrical potential (voltage).

Definitions of the variables used in circuit analysis are given below:

1. *Current.* Electrical current is usually designated with the symbol i (or I). It is defined as the time rate of change of the charge (designated q with units of coulombs). The electrical current has the units of the ampere (A).

$$i = \text{time rate of change of charge} = \frac{dq}{dt} \qquad q = \text{charge (C)}$$

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2. *Voltage.* Voltage is the electromotive force or potential difference and is designated with the symbol V (or e, E, v). It is a measure of the potential of an electric field. The electrical voltage has the units of volts and is generally written

$$V_{AB} = \text{potential of } A \text{ with respect to } B$$

or more commonly with a (plus sign) at point A and a (minus sign) at point B.

3. *Power.* Power is defined as the ability to do work. The flow of power is generally what is described by engineering models. Power has units of watts and is the product of the voltage and current.
4. *Work.* Work (or energy) has units of joules and is the integral of power over time:

$$W = \int_{t_0}^{t_f} P(t)\, \partial t \tag{1}$$

1.2 Basic Electrical Elements

Several elements and their constitutive relationships need to be understood to analyze basic electrical circuits. The constitutive relationship for each element describes the relationship between the voltage (effort variable) and the current (flow variable):

1. *Ideal Voltage Source.* The ideal voltage source produces a voltage, or electrical potential, as a function of time. This voltage produced by the source is an input to the system and the voltage source adds power to the system. The current passing through the ideal voltage source is not generally known. This current is determined by the other elements connected to the voltage source. The ideal voltage source is capable of producing infinite current and therefore infinite power. See Fig. 2.
2. *Ideal Current Source.* The ideal current source produces a current as a function of time. This current produced by the source is an input to the system and the current source adds power to the system. The voltage across the ideal current source is not generally known. This voltage is determined by the other elements connected to the current source. The ideal current source is capable of producing infinite voltage and therefore infinite power. See Fig. 3.
3. *Resistor.* The resistor restricts the flow of current and dissipates power. The magnitude of this resistance is denoted R and has the units of ohms (Ω). The resistor is a passive element in that it does not add energy to the system. The resistor is governed by a passive sign convention, as shown in Fig. 4, in that the current flows across the resistor from the side of higher potential (voltage) to the side of lower potential (voltage). If the current and the voltage are negative, this indicates that current flow is in the opposite direction and the potential is higher on the opposite side. In linear

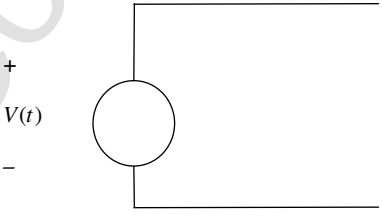


Figure 2 Ideal voltage source.

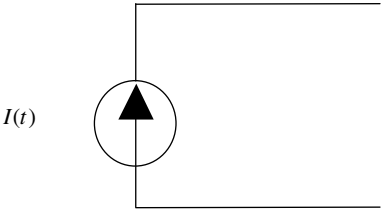


Figure 3 Ideal current source.

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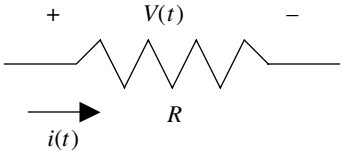


Figure 4 Ideal resistor.

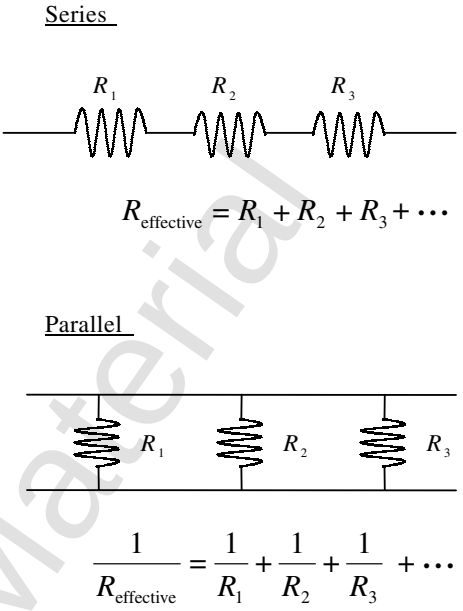


Figure 5 Resistors.

resistors the relationship between voltage and current is linear. Linear resistors are governed by Ohm's law, given by $V(t) = i(t)R$.

The magnitude of the resistance of a long cylindrical wire is related to the resistivity of the material the wire is made of (copper and gold have low resistivity) and the length of the wire. It is inversely related to the cross-sectional area of the wire:

$$R = \frac{\rho \ell}{A} = \frac{(\text{resistivity})(\text{length})}{\text{cross-sectional area}} \tag{2}$$

Series and parallel combinations of resistors can be thought of as an effective resistance. Resistors connected in series (share common current) create an effective resistance equal to the sum of the resistors. Resistors connected in parallel create an effective resistance where the inverse of the effective resistance is related to the sum of the inverse of each of the resistors. See Fig. 5.

If there are two parallel resistors, this expression becomes

$$R_E = \frac{R_1 R_2}{R_1 + R_2} \tag{3}$$

The resistance is sometimes described by the conductance G, defined as the inverse of the resistance:

$$G = \frac{1}{R} \tag{4}$$

4. *Capacitor.* The capacitor collects electrical charge and stores energy. The magnitude of the capacitance is denoted C and has the units of farads (F, or coulombs per volt). The capacitor is an energy storage element. The capacitor, like the resistor, is gov-

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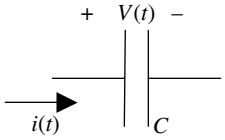


Figure 6 Ideal capacitor.

erned by a passive sign convention as shown in the figure in that the current flows "across" the capacitor from the side of higher potential (voltage) to the side of lower potential (voltage). If the current (and the voltage) are negative, this indicates that current flow is in the opposite direction and the potential is higher on the opposite side. See Fig. 6. In linear capacitors, the voltage is linearly related to the time integral of the current. Linear capacitors are governed by the equation

$$V(t) = \frac{1}{C}\int_0^t i(t)\ \partial t \tag{5}$$

or

$$i(t) = C\frac{\partial V(t)}{\partial t} \tag{6}$$

The simplest capacitor is a parallel-plate capacitor that consists of two plates separated by a nonconducting material called a dielectric. The magnitude of the capacitance for a parallel-plate capacitor is proportional to the dielectric constant of the material property between the plates and the area of the plates. It is inversely related to the distance between the plates. Such capacitors are often used as position sensors by changing either the distance between the plates or the area of overlap between the plates (see Fig. 7):

$$C = \frac{K\epsilon_0 A}{d} \tag{7}$$

where K = dielectric constant
ϵ = permeability constant
A = area of plates
d = distance between plates

Series and parallel combinations of capacitors can be thought of as an effective capacitance. Capacitors connected in series (share common current) create an effective capacitance where the inverse of the effective resistance is related to the sum of the inverse of each of the capacitances. Capacitors connected in parallel create an effective capacitance equal to the sum of the capacitances. See Fig. 8.

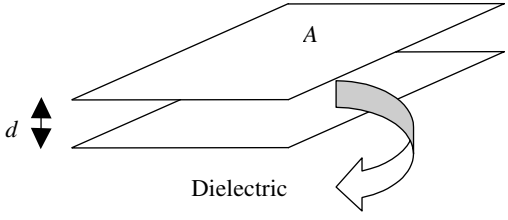


Figure 7 Parallel-plate capacitor.

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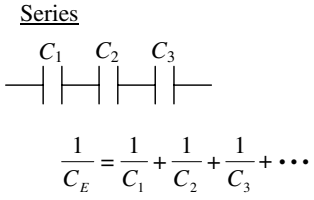


Figure 8 Capacitors.

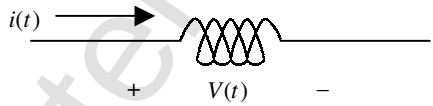


Figure 9 Ideal inductor.

5. *Inductors.* The inductor stores magnetic energy by creating a magnetic field. The magnitude of the inductor is denoted L and has the units of henrys (H). The inductor is an energy storage element. The inductor, like the resistor, is governed by a passive sign convention, as shown Fig. 9, in that the current flows through the inductor from the side of higher potential (voltage) to the side of lower potential (voltage). If the current and the voltage are negative, this indicates that current flow is in the opposite direction and the potential is higher on the opposite side. In linear inductors, the drop across the inductor, $V(t)$, is linearly related (by L) to the time rate of change of the current passing through the inductor.

Linear inductors are governed by the equations

$$V(t) = L\frac{dI(t)}{dt} \tag{8}$$

$$I(t) = \frac{1}{L}\int_0^t V(t)\,\partial t \tag{9}$$

Series and parallel combinations of inductors can be thought of as an effective inductance. Inductors connected in series (share common current) create effective inductance equal to the sum of the inductors. Inductors connected in parallel create an effective inductance where the inverse of the effective inductance is related to the sum of the inverse of each of the inductors. See Fig. 10.

6. *Ideal Diode.* The ideal diode is a nonlinear element. It does not store, add, or dissipate energy. The ideal diode can be compared to an ideal check valve in a fluidic system. The diode allows current to flow unimpeded (no voltage drop) in the positive direction. The diode allows no current to flow in the negative direction. Therefore, I can only be positive and V can only be negative, as shown in Fig. 11.

1.3 Circuit Analysis

Basic circuit analysis relies on two laws: (1) Kirchhoff's voltage law and (2) Kirchhoff's current law. There are several approaches and techniques to circuit analysis, but a straight-

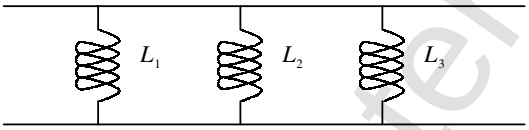
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Series



$$L_E = L_1 + L_2 + L_3 + \cdots$$

Parallel



$$\frac{1}{L_{\text{eq}}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots$$

Figure 10 Inductors.

forward application of these two laws as presented here will be effective in all situations. This straightforward application, however, may lead to some additional algebra. Experience with these problems can lead to shortcuts.

Kirchhoff's Voltage Law

$$\sum_{i=1}^{n} V_i = 0 \tag{10}$$

Kirchhoff's voltage law states that the sum of voltages around a closed loop is zero. To a mechanical engineer, this is analogous to the movement of a mass in a potential field (such as gravity). For example, it is clear that if a bowling ball is picked up, rolled down the lane, and then returned to the original position, there is no net change in the ball's potential energy

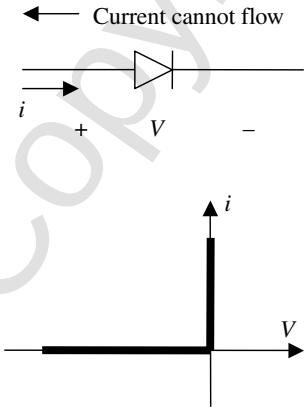


Figure 11 Ideal diode.

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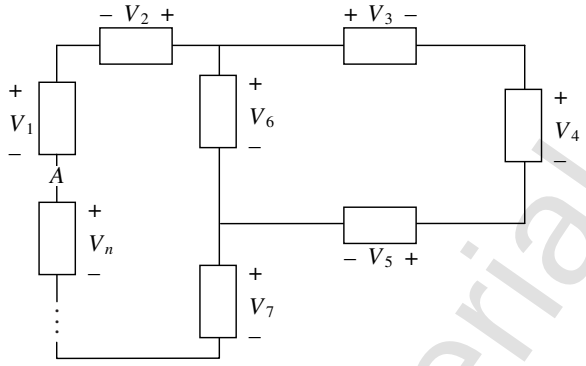


Figure 12 Kirchhoff's voltage law.

as it moved through this closed path. Similarly, Kirchhoff's voltage law states that the sum of the potential changes (voltages) around a closed loop is zero. The closed loops that can be used to formulate these equations are not unique. However, they will not all be independent (see Fig. 12):

1. Pick a starting point (e.g., *A*).
2. Choose a loop direction [clockwise (CW)].
3. Write the equation as you go around the loop:

$$V_1 + V_2 - V_3 - V_4 - V_5 - V_7 + \cdots + V_n = 0 \tag{11}$$

Kirchhoff's Current Law

$$\sum_{i=1}^{N} I_i = 0 \tag{12}$$

Kirchhoff's current law states that the net sum of the current flowing into a node is zero. To a mechanical engineer, this is analogous to fluid flow into and out of a node of a pipe network. If no fluid is stored in the node, then mass conservation requires that the mass flow rates into (positive flow) and out of (negative flow) the node must be zero; (that is, what goes in must come out) (see Fig. 13):

1. Assume current directions and draw arrows.
2. Assume current directions must be consistent with assumed voltage drops.
3. Apply Kirchhoff's current law (assume current into the node is positive):

$$i_1 - i_2 + i_3 - i_4 + \cdots + i_n = 0 \tag{13}$$

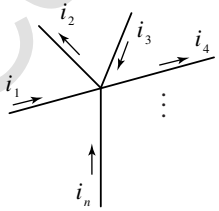


Figure 13 Kirchhoff's current law.

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1.4 Sources and Meters

Some practical considerations are important when building mechatronic systems. Sources and meters (and all elements) do not behave as the ideal representation described above. Some of the limitations are described here.

1. *Voltage Source.* An ideal voltage source as described above does not have any output resistance (no resistor shown in the above representation) and can supply infinite current. Obviously, a real voltage source does have an output resistance. The output resistance (usually $<1\ \Omega$) can be represented as the model given below. Now, as current is output from the source, the output voltage (V_{out}) is no longer the same voltage as the source voltage (V_s). The magnitude of this output resistance (and the corresponding voltage drop) changes with many factors. For example, the output resistance of a rechargeable battery (such as a car battery) increases as the battery ages. The most significant difference between an ideal voltage source and a real source is that the real source can provide only a limited current. This is a limitation on both the instantaneous current that can be produced as well as the total amount (time integral) of current that can be produced. For example, the power supply limitation on instantaneous current is often a constraint for battery-operated devices such as cell phones or robots. See Fig. 14.

2. *Current Source.* An ideal current source as described above does not have any output resistance (no resistor shown in the above representation) and can supply infinite voltage. Obviously, a real current source does have an output resistance. The output resistance (usually $>1\ \text{M}\Omega$) can be represented as the model given below. Now, as current is output from the source, the output current (I_{out}) is no longer the same current as the source current (I_s). The magnitude of this output resistance (and the corresponding voltage drop) changes with many factors. The most significant difference between an ideal current source and a real source is that the real source can provide only a limited voltage. This is a limitation on both the instantaneous voltage that can be produced as well as the total amount (time integral) of voltage that can be produced. See Fig. 15.

3. *Voltmeter.* An ideal voltmeter has infinite input resistance and draws no current from the voltage being measured. A real voltmeter has a finite input resistance as modeled below and does draw some current from the source which can change the voltage being measured. However, the input resistance of most real voltmeters is very large (usually several mega ohms) and this makes the voltmeter a very safe device as it does not draw "significant" current. See Fig. 16.

4. *Ammeter.* An ideal ammeter has zero input resistance and does not produce a voltage drop. A real ammeter has a finite input resistance as modeled below and does have a small voltage drop across the leads of the meter. However, the input resistance of most real ammeters is very small (a few ohms), and this makes the ammeter a device that requires careful consideration before use. If the ammeter leads are placed between two points with a potential difference, a short circuit will occur and very large current will be produced. Therefore, most

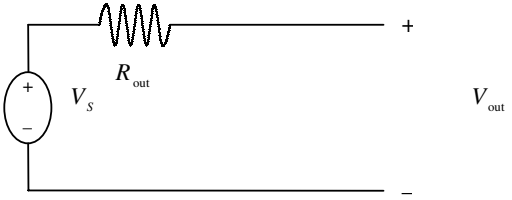


Figure 14 Real voltage source.

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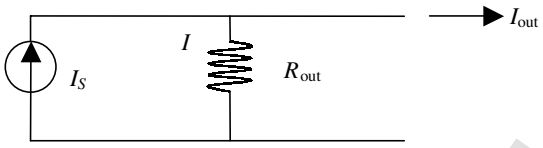


Figure 15 Real current source.

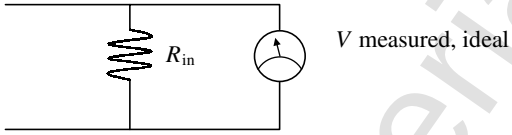


Figure 16 Real voltage meter.

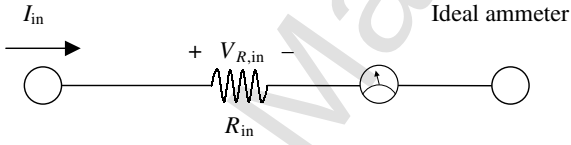


Figure 17 Real current meter.

devices that function as ammeters and voltmeters (multimeters) require that the user remove the leads and place them into new terminals before the devices are used as ammeters. This requires the user to be very deliberate about using the ammeter. See Fig. 17.

5. *Ohmmeter.* An ohmmeter is used to measure the resistance between the two terminals. The ohmmeter consists of a voltage source and an ammeter. A voltage is applied between the leads, the current is measured, and the resulting resistance is found using Ohm's law.

1.5 RL and RC Transient Response

To develop a basic understanding of the effects of inductors and capacitors in circuits, it is useful to understand *RL* and *RC* circuits. These circuits contain one resistor and one inductor (*RL*) or one resistor and one capacitor (*RC*). The equations of these circuits (developed with the laws above) are first-order differential equations with constant coefficients of the form

$$X(t) = B\frac{dy}{dt} + Cy \tag{14}$$

If we assume step inputs, a solution of the forms in Figs. 18 and 19 is assumed as solution to this equation. These first-order responses to step inputs rise or decay according to a time constant T:

1. First-order rise:

$$y(t) = Ae^{-(1/T)t} \tag{15}$$

2. First-order decay:

$$y(t) = A(1 - e^{-(1/T)t}) \tag{16}$$

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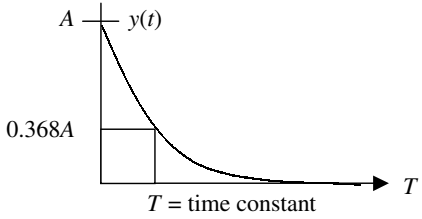


Figure 18 First-order response.

Consider the following circuits and the resulting equations, solutions, and time responses to step inputs:

1. *RL* circuits:

$$T = \frac{L}{R} \tag{17}$$

a. Differential equation:

$$Ri + L\frac{di}{dt} = V \tag{18a}$$

Solution:

$$i = \frac{V}{R}(1 - e^{-t/T}) \quad \text{(see Fig. 20)} \tag{18b}$$

b. Differential equation:

$$Ri + L\frac{di}{dt} = 0 \tag{19a}$$

Solution:

$$i = \frac{V}{R}(1 - e^{-t/T}) \quad \text{(see Fig. 21)} \tag{19b}$$

2. *RC* circuits:

$$T = RC \tag{20}$$

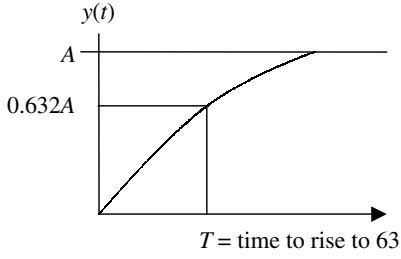


Figure 19 First-order response.

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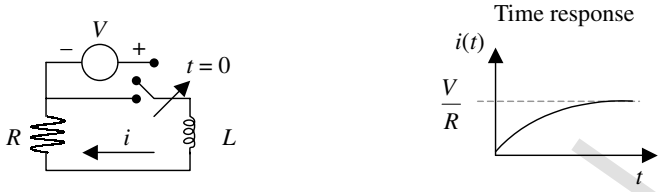


Figure 20 An *RL* circuit (see Eq. 18b).

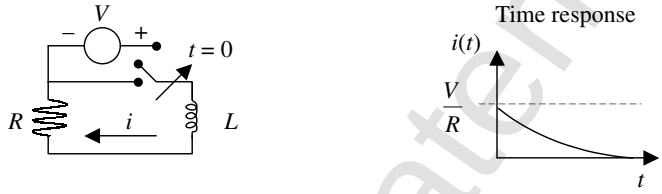


Figure 21 An *RL* circuit (see Eq. 19b).

a. Differential equation:

$$\frac{V}{R} + C\frac{dv}{dt} = I \tag{21a}$$

Solution:

$$v = RI(1 - e^{-t/T}) \quad \text{(see Fig. 22)} \tag{21b}$$

b. Differential equation:

$$\frac{V}{R} + C\frac{dv}{dt} = 0 \tag{22a}$$

Solution:

$$v = RIe^{-t/T} \quad \text{(see Fig. 23)} \tag{22b}$$

The resulting observations from these circuits are that the voltage does not change instantly across a capacitor, but the current can change instantly. Also, the voltage can change instantly across an inductor, but the current cannot change instantly:

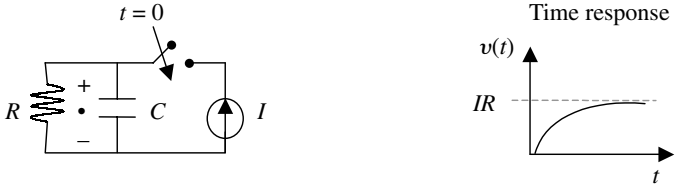


Figure 22 An *RC* circuit (see Eq. 21b).

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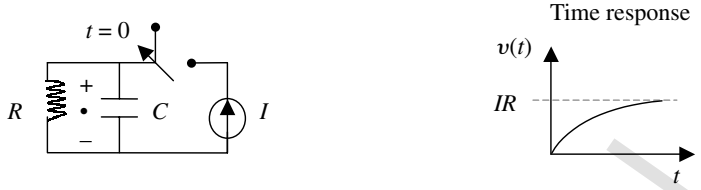


Figure 23 An *RC* circuit (see Eq. 22b).

	V	*I*
C	No	Yes
L	Yes	No

The instantaneous behavior (t is small) steady-state behavior (t is large) is also interesting. This shows that a capacitor begins like a short circuit and becomes an open circuit. Also, an inductor begins as an open circuit and becomes a short circuit:

	Capacitor	Inductor
$t = 0$	Short	Open
$t = \infty$	Open	Short

1.6 Basic Electronics Examples

Consider the basic circuit in Fig. 24. Applying Kirchhoff's current law to node *A* reveals

$$i_1 + i_2 = 1 \tag{23}$$

Now Kirchhoff's voltage law can be written as

$$500i_1 + 100i_2 - 100 - 200i_2 = 0 \tag{24}$$

and it is possible to substitute for

$$i_1 \ (i_1 = 1 - i_2) \tag{25}$$

Thus

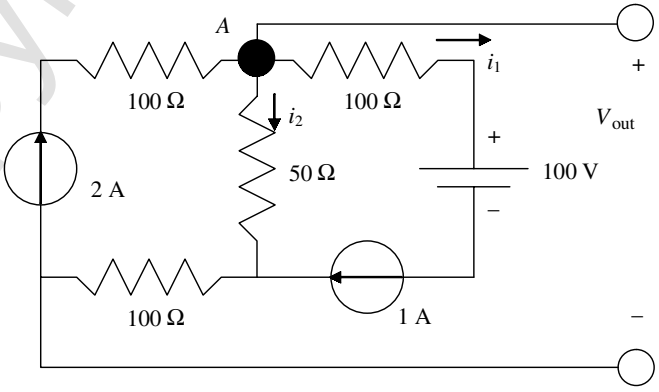


Figure 24 Example electrical circuit.

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$$500 - 500i_2 - 100i_2 - 100 - 200i_2 = 0 \tag{26}$$

$$800i_2 = 400 \tag{27}$$

where $i_2 = \frac{1}{2}$ and $i_1 = 1$. Now Kirchhoff's voltage law can be written for the outside loop:

$$V_{\text{out}} - 200 - 100i_2 - 100 = 0 \tag{28}$$

Therefore,

$$V_{\text{out}} = 350 \text{ V} \tag{29}$$

Transfer Function for an RL Circuit

Consider the circuit in Fig. 25. Kirchhoff's current law can be written as

$$I_{\text{in}} = i_1 + i_2 \tag{30}$$

Kirchhoff's voltage law can be written as

$$V_{\text{out}} + L\frac{di_2}{dt} - R_1 i_1 = 0 \tag{31}$$

and as

$$\frac{V_{\text{out}}}{R_L} = i_2 \tag{32}$$

Substituting (32) into (31) gives

$$V_{\text{out}} + \frac{L}{R_L}\frac{dV_{\text{out}}}{dt} - R_1\left(I_{\text{in}} - \frac{V_{\text{out}}}{R_L}\right) = 0 \tag{33}$$

The transfer function can be found using La Place transformation:

$$V_{\text{out}}(s) + \frac{L}{R_L}sV_{\text{out}}(s) - R_1 I_{\text{in}}(s) + -\frac{R_1}{R_L}V_{\text{out}}(s) = 0 \tag{34}$$

Rearranging gives

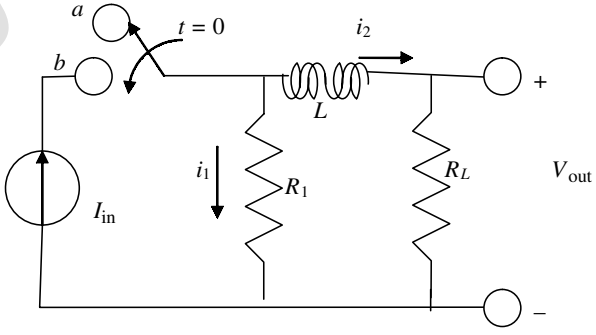


Figure 25 Example electrical circuit.

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$$\frac{V_{\text{out}}(s)}{I_{\text{in}}(s)} = \frac{R_1}{(L/R_L)\ s + (1 + R_1/R_L)} \tag{35}$$

2 OPERATIONAL AMPLIFIERS

Operational amplifiers, or op amps, are extremely useful in mechatronics. They are primarily used to modify an analog signal, such as increasing the voltage output of a strain gauge circuit or removing noise from a signal. They are often used to amplify, sum, subtract, integrate, or differentiate analog signals. They are also used to create active filters. They are active devices in that they add energy to the circuit.

Consider the ideal op amp model in Fig. 26. All voltages shown in the figure are referenced to the same ground. As active elements, the op amp adds energy to the circuit through the power supplied by $+V_s$, $-V_s$ (V_s is often but does not have to be 15 V). The supply voltages are often left off Fig. 26 for convenience.

Circuit analysis of op amps requires only two additional observations. For ideal op amp behavior it must be remembered that

$$\Delta V = (V_+ - V_-) = 0 \tag{36}$$

or

$$V_- = V_+ \tag{37}$$

and

$$i_- = i_+ = 0 \tag{38}$$

It should also be noted that, in general,

$$i_{\text{out}} \neq 0 \tag{39}$$

Op amps produce an output voltage (V_{out}) that is related to an input voltage (V_{in}, not necessarily V_+ or V_-). The relationship can frequently be viewed as a gain defined by

$$A_v = \frac{V_{\text{out}}}{V_{\text{in}}} \tag{40}$$

The op amp generally has a very high input impedance (ideally infinite) defined by

$$Z_{\text{in}} = \frac{V_{\text{in}}}{i_{\text{in}}} \tag{41}$$

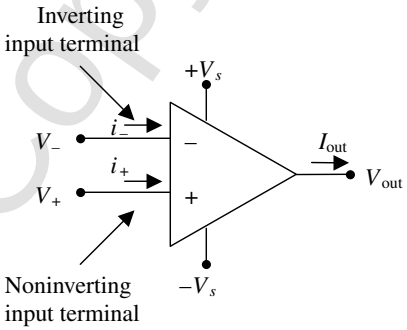


Figure 26 Ideal operational amplifier.

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For real op amps the input impedance is usually >100 kΩ. Op amps also have a very low output impedance (ideally zero) defined by

$$Z_{\text{out}} = \frac{V_{\text{out}}}{i_{\text{out}}} \tag{42}$$

For real op amps this is usually less than a few ohms.

Circuits for two of the most common op amps, the inverting op amp and the noninverting op amp, are analyzed in Figs. 27 and 28.

2.1 Inverting Amplifier

Apply op amp rules:

$$\Delta V = 0 \tag{43}$$

or

$$V_- = V_+ \tag{44}$$

$$i_- = i_+ = 0 \tag{45}$$

Kirchhoff's current law gives

$$i_R = i_{Rf} = i \tag{46}$$

Kirchhoff's voltage law gives

$$V_{\text{in}} - i_{\text{in}}R = 0 \tag{47}$$

$$\frac{V_{\text{in}}}{R} = i_{\text{in}} \tag{48}$$

$$V_{\text{out}} + i_{\text{out}}R_f = 0 \tag{49}$$

$$-\frac{V_{\text{out}}}{R_f} = i_{\text{out}} \tag{50}$$

Setting $i_{\text{in}} = i_{\text{out}}$ gives

$$\frac{V_{\text{in}}}{R} = \frac{-V}{R_f} \tag{51}$$

or

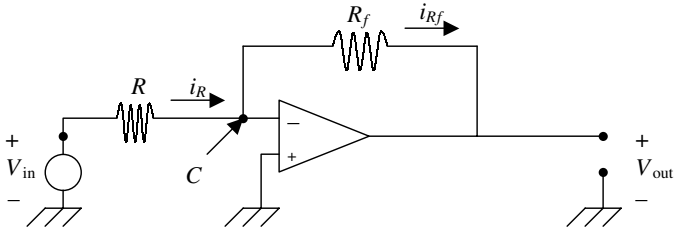


Figure 27 Inverting operational amplifier.

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$$\frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{R_f}{R} \tag{52}$$

Notice that for the inverting amplifier the gain is negative—hence the name ''inverting.'' This means that for periodic signals there would be a 180° phase shift between the output voltage and the input voltage. The gain for this circuit is

$$A = -\left(\frac{R_f}{R}\right) \tag{53}$$

$$V_{\text{out}} = AV_{\text{in}} \tag{54}$$

2.2 Noninverting Amplifier

Kirchhoff's current law yields

$$i_{\text{in}} = i_{\text{out}} \tag{55}$$

since $i_+ = i_- = 0$. Kirchhoff's voltage law yields

$$V_{\text{in}} + Ri = 0 \tag{56}$$

$$V_{\text{out}} + R_f i - V_{\text{in}} = 0 \tag{57}$$

$$i = -\frac{V_{\text{in}}}{R} \tag{58}$$

$$i = \frac{V_{\text{in}} - V_{\text{out}}}{R_f} \tag{59}$$

Setting the i's equal gives

$$-\frac{V_{\text{in}}}{R} = \frac{V_{\text{in}} - V_{\text{out}}}{R_f} \tag{60}$$

Dividing by V_{in} yields

$$-\frac{R_f}{R} = 1 - \frac{V_{\text{out}}}{V_{\text{in}}} \tag{61}$$

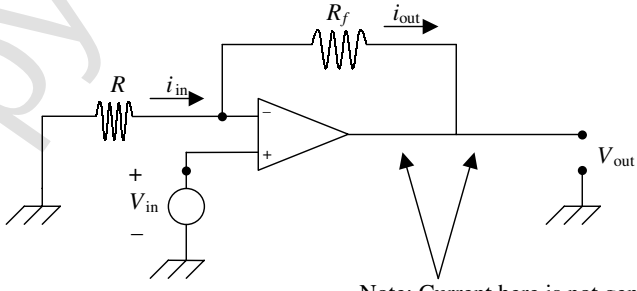


Figure 28 Noninverting operational amplifier.

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$$\frac{V_{\text{out}}}{V_{\text{in}}} = \underbrace{1 + \frac{R_f}{R}}_{\text{Gain} \geq 1} \tag{62}$$

Here V_{out} and V_{in} have the same sign so the amplifier is noninverting. This circuit is often used as a buffer to "isolate" one portion of a circuit from another.

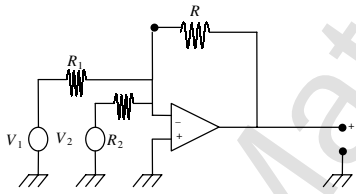
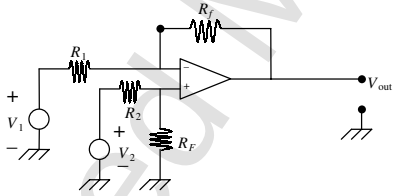
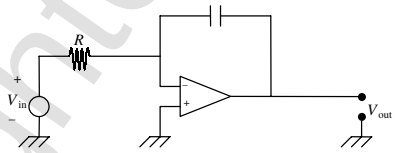
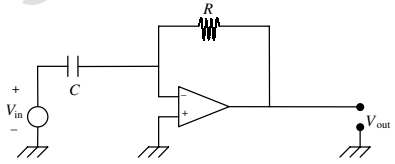
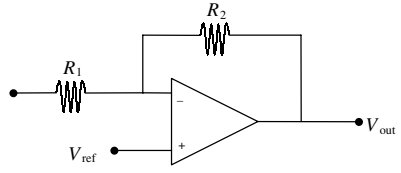
Type	Schematic	Input/Output Equation
Summer		$V_{\text{out}} = -(V_1 + V_2)$
Difference amplifier		$V_{\text{out}} = \frac{R_f}{R}(V_2 - V_1)$
Integrator		$V_{\text{out}}(t) = -\frac{1}{RC}\int_0^t V_{\text{in}}(x)\,dx$
Differentiator		$-RC\left(\frac{dV_{\text{in}}}{dt}\right) = V_{\text{out}}(t)$
Gain and shift amplifier		$V_{\text{out}} = -\frac{R_2}{R_1}V_1 + \left(1 + \frac{R_2}{R_1}\right)V_{\text{ref}}$ if $R_1 = R_2 = R$ $V_{\text{out}} = \frac{R_f}{R}(V_2 - V_1)$

Figure 29 Useful op amp circuits.

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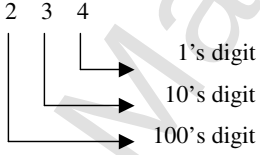
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2.3 Other Common Op Amp Circuits

Some other useful op amp circuits are described in Fig. 29 (page 843).

3 BINARY NUMBERS

Computers and digital electronics used in mechatronic systems are described by binary arithmetic. Therefore, it is important to understand binary numbers to fully understand the function of computers. First, consider a base-10 number as in standard mathematics. Base-10 numbers have 10 possible digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9). Consider a number such as 234. Here the 4 represents the 1's digit, the 3 represents the 10's digit, and the 2 represents the 100's digit. The number 234 represents four 1's and three 10's and two 100's ($4 * 1 + 3 * 10 + 2 * 100 = 234$). The value of each digit increases by a factor of 10 as you move to the left and decreases by a factor of 10 as you move to the right. Consider a number such as 234:



4 represents the sum of nine 1's

3 represents the sum of seven 10's

2 represents the sum of one 100

Now, consider a base-2 number, or a binary number. Base-2 numbers have two possible digits (0, 1). Again, consider the number 234. With binary numbers the value of each digit increases by a factor of 2 as you move to the left and decreases by a factor of 2 as you move to the right. To write this number as a binary number requires zero 1's and one 2's and zero 4's and one 8's and zero 16's and one 32's and one 64's and one 128's ($234 \equiv 11101010 = 0 * 1 + 1 * 3 + 0 * 4 + 1 * 8 + 0 * 16 + 1 * 32 + 1 * 64 + 1 * 128$):

Binary base 2: two possible digits (0, 1)

$$234 \equiv 11101010$$

0	1's
1	2's
0	4's
1	8's
0	16's
1	32's
1	64's
1	128's

3.1 Binary Numbers of Different Size

Each digit in a binary number (either a 0 or a 1) is called a *bit*—"binary digit." A nibble is a group of 4 bits, a byte is a group of 8 bits, a word is a group of 16 bits, and a double word (dword) is a group of 32 bits. Each bit is numbered starting with zero and moving to

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the left. The K^{th} bit represents the 2^K place holder. The zeroth bit is called the least significant bit (LSB) and the highest bit (e.g., seventh in illustration) is called the most significant bit (MSB):

Identifying individual bits (see Fig. 30):

- Starting from the right
- The Kth bit represents the 2^K slot

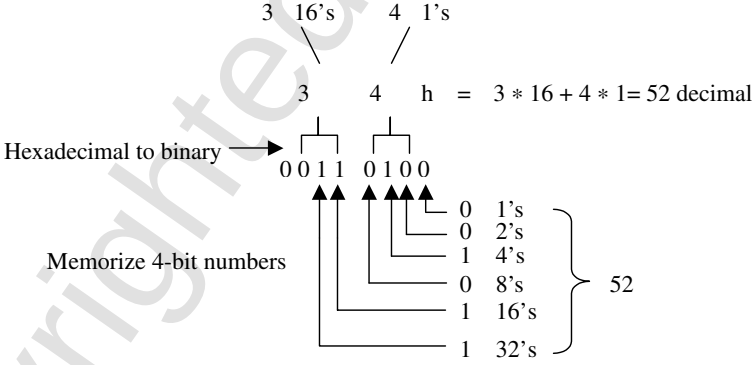
Least significant bit: bit furthest to the right

Most significant bit: bit furthest to the left

3.2 Hexadecimal Numbers

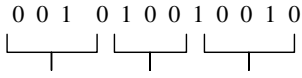
Hexadecimal numbers are binary numbers that are easier for humans to work with. Hexadecimal numbers have 16 unique digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F). Since hexadecimal numbers sometimes use both letters and numbers an "h" is usually placed at the end of the number to distinguish it as a hexadecimal number (e.g., ACEh). Again, the value of each digit increases by a factor of 16 as you move to the left and decreases by a factor of 16 as you move to the right. The advantage of hexadecimal numbers is that each hexadecimal digit represents a 4 bit binary number. This makes it very simple to convert from hexadecimal to binary (see Fig. 31).

Example 1 34h = 52:



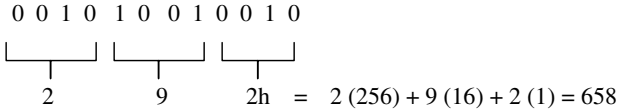
52 decimal = 34h = 0 0 1 1 0 1 0 0

Example 2 Binary to hexadecimal:



1. Break into 4-bit segments (add two 0's to left).

2. Convert each 4-bit segment into one hexadecimal digit using Fig. 31:



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7th bit			*K*th bit		2nd bit	1st bit	0th bit
1	0	1	1	0	0	1	1

Figure 30 A byte.

Hexadecimal	Binary
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

Figure 31 Hexadecimal-to-binary conversion.

3.3 Binary Addition

Binary addition is straightforward and similar to decimal addition. The two numbers are first aligned by their respective columns. Then the 1's digits are added. If the result is greater than the maximum allowable number for that column (i.e., 1), the remainder is carried into the next column. This process is repeated until all columns are added and no remainder is left:

$$\begin{array}{r} 3 \\ +\ 1 \\ \hline 4 \end{array} \qquad \begin{array}{r} 011 \\ +\ 001 \\ \hline 100 \end{array}$$

3.4 Two's Complement Binary Numbers

Straight (regular) binary notation cannot represent a negative numbers. A new convention called two's complement binary numbers is used. Then binary arithmetic can be used to add a positive number to a negative number and in this way it is possible to perform binary subtraction. Consider a 3-bit two's complement example:

(3-Bit Example)		For N bits
011	3	2^{N-1}
010	2	
001	1	
000	0	0
111	−1	
110	−2	
101	−3	
100	−4	-2^{N-1}

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The procedure to convert from a positive number (e.g., 3) to the negative of that number (e.g., −3) is:

(i) Invert all bits.

(ii) Add 1 to the result.

The procedure to convert from a negative number (e.g., −3) to the positive of that number (e.g., 3) is:

(i) Subtract 1 (add −1).

(ii) Invert all bits.

Consider the following 3-bit examples:

Example 3 3 bits

To change a positive 3 (written as 011 in 3-bit binary) to a negative 3

(i) Invert all bits:

$$100$$

(ii) Add 1:

$$\begin{array}{r} 100 \\ \underline{001} \\ 101 \Rightarrow -3 \end{array}$$

Example 4 To convert a 3-bit two's compliment negative 2 to a positive 2.

(i) Subtract 1:

$$\begin{array}{r} 110 \\ \underline{111} \\ 101 \end{array}$$

(ii) Invert all bits:

$$010 \Rightarrow 2$$

Now two two's complement numbers can be added and this is like binary subtraction. Consider the following example:

$$\begin{array}{rl} (-3) + (2) \longleftrightarrow & 101 \\ & \underline{010} \longleftrightarrow -1 \\ & 111 \end{array}$$

4 DIGITAL COMPUTERS

Mechanical engineers should be familiar with the basic low-level operations of microcontrollers and computers so as to (1) understand what computers can and cannot do in mechatronic systems, (2) understand how computers are used in mechatronic systems (what their job is), (3) be able to communicate with electrical and computer engineers on interdisciplinary design teams, and (4) be able to select a microcontroller when designing a mechatronic system. With these stated goals only the general concepts are presented here. The details of low-level computer operation are numerous and beyond the scope of this chapter.

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	(MSB)							(LSB)
Bit #	7	6	5	4	3	2	1	0
	X	X	X	X	X	X	X	X

where *x* can be a 0 or a 1.

Figure 32 Register.

4.1 Most Basic Computer

The most basic computer includes a central processing unit (CPU) and some amount of memory. The memory holds numbers while the CPU, sometimes called the microprocessor, holds numbers and processes these numbers. The CPU sends some control information (including addresses) to the memory and the CPU and memory exchange data. The CPU holds and processes data in registers. A register is a set of flip-flops (a digital electronic device with two states—0, 1) whose contents are read or written as a group. Registers come in several sizes such as 8, 16, and 32 bits. Consider the 8-bit register shown in Fig. 32, where x can be a 0 or a 1.

Registers are used for temporary storage of numbers and manipulation of numbers (addition, subtraction, etc.).

4.2 Memory

In almost all computers, memory is arranged as a series of bytes. Each memory location can hold a single 8-bit binary number (00000000 to 11111111). More than one memory location is needed to store numbers larger than 255. Memory can be thought of as an 8-bit register, as shown above. Memory can come in many forms, such as solid-state electronic chips [random-access memory (RAM)], magnetic storage devices such as floppy and hard disks, and optical devices such as CDs and DVDs. A computer usually has thousands, millions, or many, many more memory locations. For example:

- 1 kb of memory is $2^{10} = 1024$ memory locations
- 64 kb of memory is $64 \times 2^{10} = 2^{16} = 65{,}563$ memory locations
- 1 Mb of memory is 1024 kb $= 2^{20} = 1{,}048{,}576$ memory locations
- 1 GB of memory is 1024 Mb $= 2^{30} = 1{,}073{,}741{,}824$ memory locations

One of the most important concepts in understanding computer memory is that *memory has an address and contents.* The address is a fixed number that is a unique identifier for a specific memory location. The contents (a single byte of data) of a specific address can be changed (see Fig. 33).

To get data from memory, the CPU loads the memory address on a special register, then sends that address to memory. Memory returns the contents of that address to another register on the CPU. The size of the number needed for the address can often be a limitation of the amount of memory a computer can have. For example, most modern desktop computers have 32-bit registers. A single register can hold a number up to 2^{32}, or 4 Gb, or 4,294,967,296. This is the number of memory locations that can be addressed with a single register (although computers often use more than one register for a memory address).

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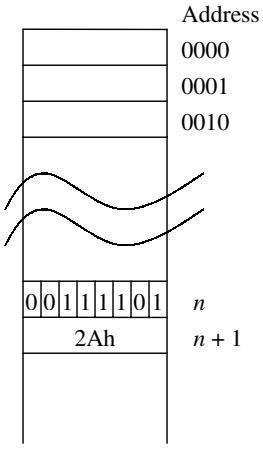


Figure 33 Schematic of computer memory.

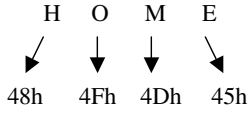
Storing Information In Memory

Each memory location always contains a number between 0 and 255. This number (along with the contents of other memory locations) can be interpreted in many ways to mean many different things:

1. *Binary Numbers.* Memory can store only unsigned binary numbers.

2. *Two's Complement Binary Numbers.* Memory can store signed binary numbers in the form of two's complement binary numbers.

3. *ASCII.* ASCII stands for the American Standard Code for Information Interchange. ASCII code is a format for characters such as letters (both upper- and lowercase), digits 0–9, and punctuation symbols. ASCII only has 128 possible symbols, so only a 7-bit binary number is needed. Since memory comes in 8-bit locations, an extra bit is added to the MSB. Consider the following example:



Since ASCII requires only 7 bits, the eighth bit is wasted. For this and several reasons there has been other formats similar to ASCII that encode more information. For example, a format used by the American National Standards Institute (ANSI) includes accented characters and a few Greek letters. Another is UNICODE, which uses 16 bits and has 65,536 different symbols.

4. *Floating-Point Numbers.* Floating-point numbers are stored in several memory locations in scientific notation. For example, 1749 must first be written in scientific notation as 1.749×10^3. There are several different types of floating-point numbers. Single-precision floating-point numbers use 32 bits (23 bits for the significant figures 1.749, 1 bit for the sign on these figures, and 8 bits for the exponent and sign on the power of ten). Single-precision numbers can range from 10^{-38} to 10^{38} with only 6–8 digits of accuracy. Double-precision floating-point numbers use 64 bits (53 bits for the significant figures with the sign and 11

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bits for the exponent and sign). Double-precision numbers can range from 10^{-308} to 10^{308} with 13–16 digits of accuracy.

4.3 Microcomputers

Microprocessors often refer to the CPU described above. Most microprocessors contain some memory in the form of registers. Microcomputers contain microprocessors connected to other devices by a common data bus. The data bus is a series of conductors that allow information to flow between each subsystem. The other devices can include input/output (I/O) devices such as monitors and keyboards or additional memory devices or I/O devices such as analog-to-digital (A/D) converters. A microcontroller is just enough microcomputer on a chip to do a specific control job. For example, a microcontroller could be a microprocessor with some digital I/O to control a home security system.

5 TRANSFER OF DIGITAL DATA

Digital data can be transferred in several ways. The most common methods are parallel and serial data transfer. Parallel communication transfers several bits at the same time along separate conductors. In serial communication data bits are sent one after the other along a single conductor. There are two types of serial communication: synchronous and asynchronous.

5.1 Parallel Data Transfer

In parallel data transfer all bits occur simultaneously on a set of data lines. Each bit is placed on each data line by the transmitting digital system and then can be read by the receiving digital system. The advantage of parallel communication is that it is relatively fast (relative to serial communication) because several bits of data can be transferred at the same time. The disadvantage is that several conductors are required. See Fig. 34.

5.2 Serial Data Transfer

In serial data transfer a sequence of bits, or train of pulses, occur on a single data line, as shown in Fig. 35.

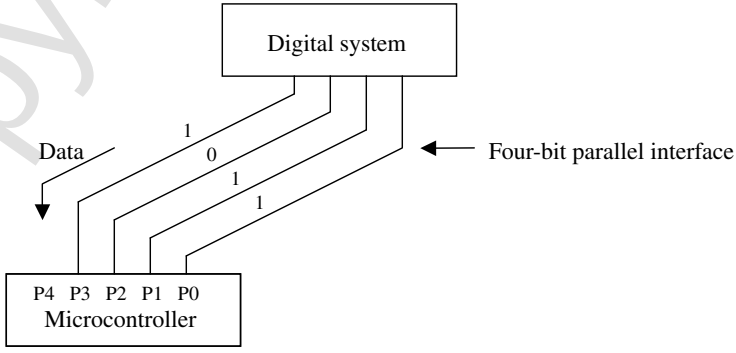


Figure 34 Four-bit parallel transfer of digital data.

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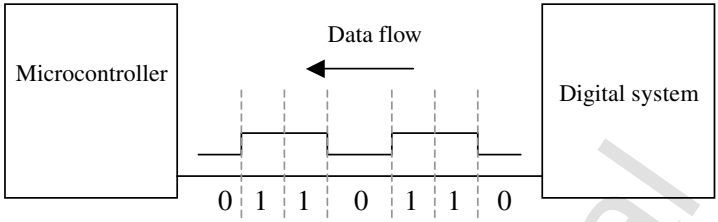


Figure 35 Serial transfer of digital data.

Asynchronous Serial Data Transfer

In asynchronous serial communication the data are written to the serial line at a predefined rate. Both the transmitter and receiver must be set for identical timing. This timing rate is called bits per second (bps) or baud rate (e.g., 4800 baud or 9600 baud). The advantage of asynchronous serial communication is that it only requires one wire (and ground) to communicate the data. This method is used in RS-232 (com ports on PCs) communication. The microprocessor detects the first "edge" of the first data bit and then (because of the baud rate) it knows how long to wait for the next bit. The baud rate is not exactly the rate the data will be transferred because there are some overhead bits required for the transfer (e.g., parity). See Fig. 36.

Synchronous Serial Data Transfer

In synchronous serial communication the data are written to the serial line and a separate line is used as a clock, or signal, to indicate the data are ready to be transferred. In this case the rate of data transfer is controlled by the digital device that provides the clock. The advantage is that the rate of transfer can be directly controlled and the transmitter and receiver do not require precise coordination. The disadvantage is that an extra line (clock) is required. See Fig. 37.

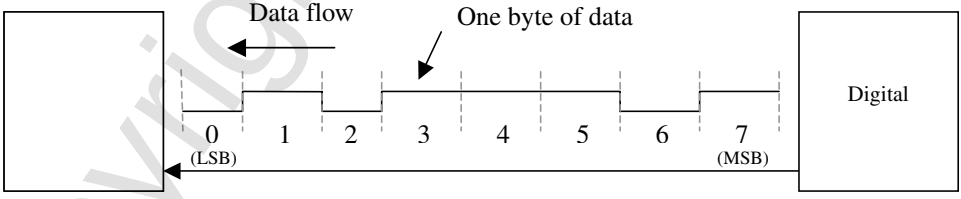


Figure 36 Asynchronous serial data transfer.

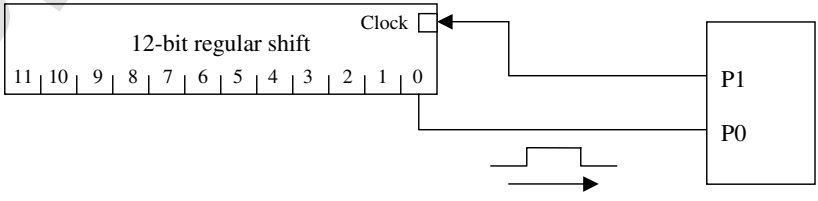


Figure 37 Synchronous serial data transfer.

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6 ANALOG-TO-DIGITAL CONVERSION

The world we live in is analog. To interface digital computers to an analog mechanical system, an A/D conversion process is required. Converting from an analog signal to a digital number is a two-step process that involves (1) quantization and (2) coding. The process of quantization is where the range of the analog signal is broken down into a discrete number of bins. In coding, the bin ''location'' of the analog signal is converted into a digital number that can be understood by the computer. There are literally hundreds of A/D converter types.

Several definitions are required to discuss A/D converters:

Resolution n refers to the number of bits used to digitally approximate the analog value of the input.

Number of possible states $N = 2^n$.

Analog quantization size Q is a measure of the analog change that can be resolved (minimum error):

$$Q = \frac{V_{\max} - V_{\min}}{N}$$

6.1 Four-Bit A/D Converter

To understand the terms explained above, the example in Fig. 38 will be used. In the example, a tachometer is used to measure the speed of an electric motor. The tachometer produces a voltage between −12 and +12 V that is linearly proportional to the speed of the motor. The analog voltage output of the motor will be read with a 4-bit A/D converter. Therefore, the A/D converter has a resolution of 4 bits and can have 16 (2^4) possible states (0000–1111 or 0–15). The analog quantization size will be 1.5 V [(12 − (−12))/16]. This means, for example, that all voltages between 10.5 and 12 V will be represented by the encoded digital number 1111.

7 DIGITAL-TO-ANALOG (D/A) CONVERTER

A D/A converter has many of the same issues as an A/D converter. The resolution is given in the number of bits used to create the output analog voltage. For example, a 4-bit D/A

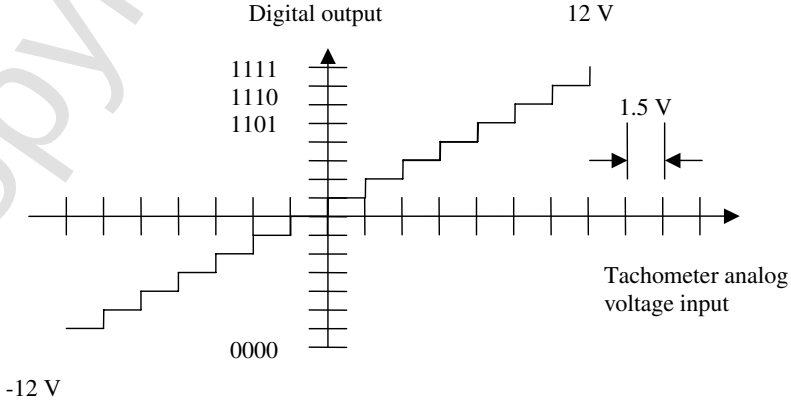


Figure 38 Analog-to-digital conversion.

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converter can produce 16 different output states. Issues to consider closely when selecting a D/A converter include the resolution and the amount of current that can be supplied.

8 SENSORS

8.1 Position Sensors

Potentiometers

One of the most basic position sensors is the potentiometer. A potentiometer is a device that generally has a mechanical input in the form of a rotary shaft or a linear slide. The mechanical input moves a wiper across a resistor. The potentiometer also has an electrical input that places a fixed voltage across the resistor. As the position of the mechanical input is changed, the location of the wiper and the voltage of an output electrical lead attached to the wiper change. The change in voltage is linearly related to the angular change of the mechanical input. Linear potentiometers that use a slide input instead of a shaft are also common.

Potentiometers are traditionally used in many applications, such as the volume knob on a radio. See Fig. 39.

Digital Optical Encoder

A digital optical encoder is another common position sensor (both rotary and linear are available) that provides a digital output. The encoder consists of a disk attached to a shaft that is the mechanical input to be measured (θ). The disk contains several ''tracks'' of alternating slots. One side of the disk has a light source and the opposite side of the disk has a photosensor. As the disk rotates, the slots alternately either block the light or allow it to pass to the photosensor. In this way the digital encoder ''encodes'' the position of the disk as either a 0 (no light) or a 1 (light). See Fig. 40.

In this way it is possible to count the alternating regions of light and dark and determine the angular change in the position of the disk/shaft. Several observations can be made from this principle. First, with only one row (track) of slits it is not possible to determine the direction the disk is rotating. Also, all measurements are made relative to a starting point. Finally, the encoder has a finite number of slits, limiting the resolution of the sensor.

These limitations are eliminated in different ways with different types of encoders. A quadrature optical encoder (described in the next section) uses two rows of slits to determine the direction of travel. Other encoders use many tracks so that each position of the shaft is unique, allowing for an absolute measurement of shaft position.

Quadrature Optical Encoder

A quadrature optical encoder includes two rows of slits that are placed 90° (a quarter cycle) out of phase. This allows the encoder to measures relative position and direction. Consider the diagram in Fig. 41, which presents the two tracks (*A* and *B*) with the encoder shaft

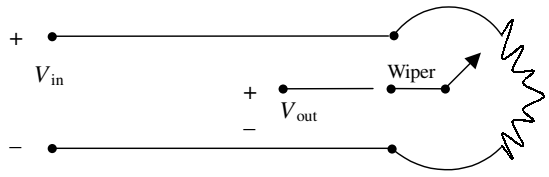


Figure 39 Potentiometer.

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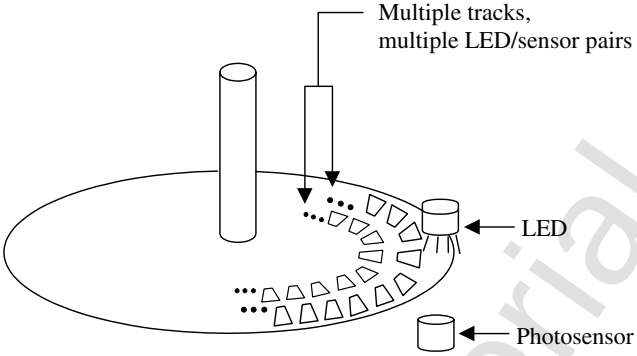


Figure 40 Rotary optical encoder.

rotating at constant velocity. Figure 41 presents the alternating regions of light and dark created by the two rows of slits. Since the slits are one-quarter cycle out of phase, there are four states created: (1) *A* on, *B* off; (2) *A* on, *B* on; (3) *A* off, *B* on; and (4) *A* off, *B* off. The direction of rotation can be determined by the order that these states appear:

$\underline{A}B$ *A* on, *B* off
$A\underline{B}$ *A* and *B* on
$\underline{A}\underline{B}$ *A* off, *B* on
$A\underline{B}$ *A* and *B* off

Direction → Output $A\underline{B}$ AB $\underline{A}B$ $\underline{A}\underline{B}$ $A\underline{B}$ $AB\ldots$

Direction ← Output $A\underline{B}$ $\underline{A}\underline{B}$ $\underline{A}B$ AB $A\underline{B}$ $\underline{A}\underline{B}$

You can get the direction by noting the change that takes place.

Linear Variable Differential Transformer

Another common position sensor is the linear variable differential transformer (LVDT). This sensor is based on an electrical transformer depicted in Fig. 42.

The transformer consists of a primary coil of wire wrapped around an iron core with a secondary coil also wrapped around the same core. As an alternating current is passed through the primary core, a changing magnetic field is created by the coil. The magnetic

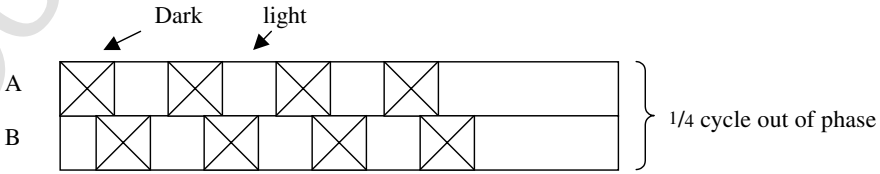


Figure 41 Output of quadrature optical encoder.

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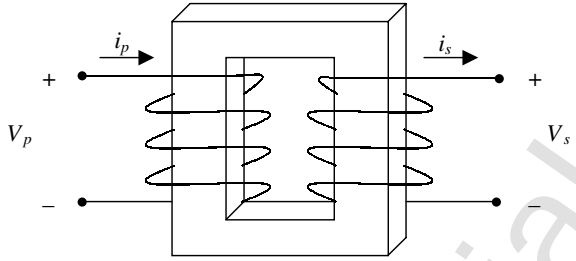


Figure 42 Electrical transformer.

field is then transferred through the high-permeability iron core. The changing magnetic filed induces a current in the secondary coil. The ideal inductor neither adds nor dissipates energy and power is conserved between the primary and secondary coils ($P_{\text{in}} = P_{\text{out}}$). The current induced (and hence the voltage) in the secondary coil is related to the current in the primary coil by the ratio of turns of each coil ($V_p/n_p = V_s/n_s$ or $n_p i_p = n_s i_s$). This principle is used (in a slightly different arrangement) to produce a position sensor.

The LVDT arrangement in Fig. 43 shows how moving the iron core (x direction) will change the coupling between the primary and secondary coils as described above. This arrangement produces a linear region where the magnitude of the ac output voltage ($=V_s$) is linearly related to the position of the movable iron core.

8.2 Force Sensors

Strain Gauge

It is impossible to directly measure a force due to the principle of causality. The effect of a force is often measured and then the force is estimated based on its effect. Strain gauges are frequently used to measure the surface stress on a structure and then estimate the force.

Strain gauges are generally a foil material where resistance changes as the foil is deformed. Strain gauges are generally glued to the surface of a structure and force is measured along a single measuring axis.

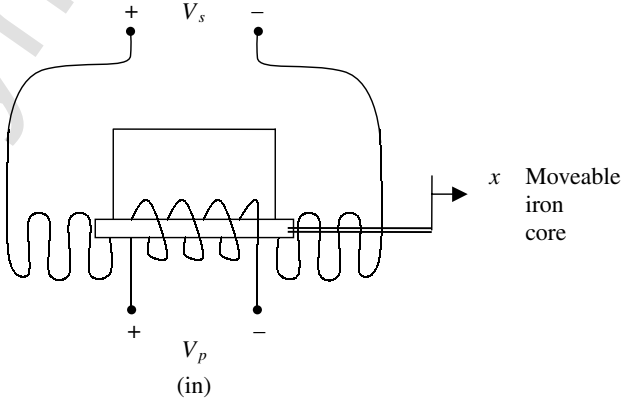


Figure 43 Schematic of LVDT.

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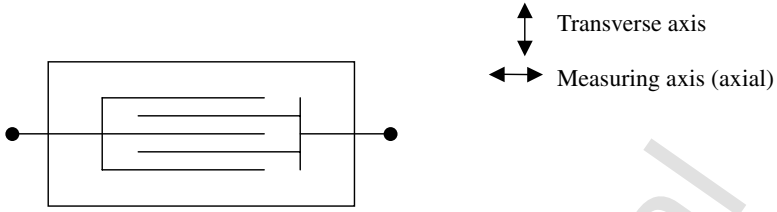


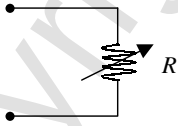
Figure 44 Schematic of strain gauge.

Strain gauges can be shown electrically as variable resistors where the resistance is related to the force. The principle is based on the physics of a long square rod, as shown in Fig. 44. The electrical resistance of the rod (R) is related to the resistivity of the material and the length of the rod and inversely related to the cross-sectional area of the rod. As a force is applied along the axis of the rod, its length (and cross-sectional area, related by Poisson's ratio) will change and thereby change the resistance. Strain gauges are designed to take maximum effect of this basic principle. It should also be noted that resistivity can change depending on many factors, including temperature. Much effort is put into advanced strain gauge applications to make the gauges independent of factors such as temperature variation. See Fig. 45.

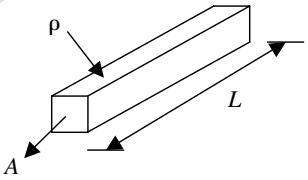
Strain gauges are described by the gauge factor, which relates the change in resistance normalized by the zero load resistance of the gauge to the strain along the axis of the gauge:

$$F = \frac{\Delta R/R}{\epsilon_{\text{axial}}} \quad (\text{often} \approx 2) \tag{63}$$

Consider the cantilevered aluminum beam example given below. The strain gauge is mounted in the center of the beam 1 in. from the end and 1 in. from the wall. The beam is 2 in. long, 1 in. wide, and $\frac{1}{8}$ in. thick. A 5-lb load is applied at the end of the beam and the gauge factor is 2. Figures 46 and 47 show a mathematical model of the beam and the strain is estimated. This is then translated into the expected change in resistance of the gauge through the gauge factor.



Strain gauge schematic
R changes with strain



$$R = \frac{\rho L}{A}$$

ρ = resistivity
(material property that can change with temperature, etc.)

L = length

A – cross-sectional area

Figure 45 Basic resistance.

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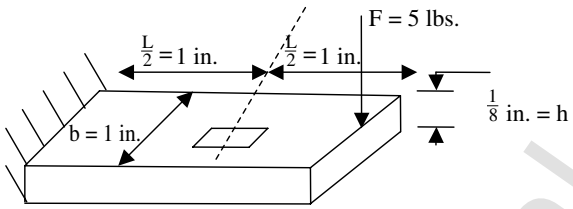


Figure 46 Strain gauge example.

Stress Modulus

$$\sigma = E\epsilon = \text{stress} = (\text{modulus of elasticity})(\text{strain}) \tag{64}$$

$$\sigma_{\text{bending}} = \frac{MC}{I} = \frac{(\text{bending moment})(\text{distance from neutral axis})}{(\text{moment of inertia})} = \text{bending stress} \tag{65}$$

$$E_{\text{Al}} = 10 \times 10^6 \text{ lb/in.}^2 \text{ (psi)} = \text{modulus of elasticity for aluminum} \tag{66}$$

$$M = F\frac{L}{2} = (5 \text{ lbs})(1 \text{ in.}) = 5 \text{ in.}\cdot\text{lb} = \text{bending moment} = \text{force}\,\frac{\text{length}}{2} \tag{67}$$

$$c = \tfrac{1}{16} \text{ in.} \tag{68}$$

$$I = \frac{1}{12}bh^3 = \frac{1}{12}\,1\,\frac{1^3}{8} = 1.6 \times 10^{-4} \text{ in.}^4 = \frac{1}{12}(\text{base})(\text{height})^3 \tag{69}$$

$$\epsilon = \frac{(5 \text{ in.}\cdot\text{lb} \times 0.0625 \text{ in.})/1.6 \times 10^{-4} \text{ in.}^4}{10 \times 10^6 \text{ lb/in.}^2} = .000192$$

$$= 192\mu\epsilon \text{ (unitless)} \tag{70}$$

$$\text{Strain} = \epsilon = 1.92\mu\epsilon \tag{71}$$

$$\text{Gauge factor} = \frac{\Delta R/R}{\epsilon_{\text{axial}}} = Z \tag{72}$$

$$\Delta R = RFE = (120)(2)(0.000192) = 0.046\ \Omega \tag{73}$$

This change in resistance is very small. It must be amplified in some way to be able to practically measure the change. If the above strain gauge is used in a voltage divider, (1)

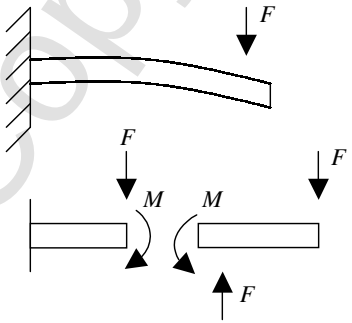


Figure 47 Free-body diagram.

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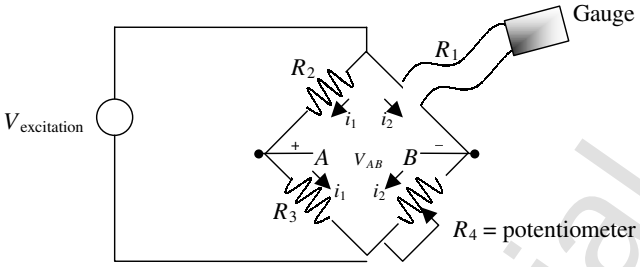


Figure 48 Strain gauge circuit.

the change in voltage will be small and (2) the voltage will be centered about a larger unchanged voltage.

For these reasons a Wheatstone bridge is used. It is shown in the analysis below that the change in voltage (V_{AB}) resulting from the change in the gauge is centered about 0 V and can then be amplified using a differential amplifier (described earlier in this chapter).

- used to measure small changes in resistance (Fig. 48).

Turn potentiometer until

$$V_A = V_B \tag{74}$$

Kirchhoff's voltage law gives

$$V_{AB} + i_1R_2 - i_2R_1 = 0 \tag{75}$$

$$V_{AB} - i_1R_3 + i_2R_4 = 0 \tag{76}$$

and

$$V_{ex} - i_1R_2 - i_1R_3 = 0 \tag{77}$$

$$V_{ex} - i_1(R_2 + R_3) = 0 \tag{78}$$

where

$$i_1 = \frac{V_{ex}}{R_2 + R_3} \tag{79}$$

and

$$V_{ex} - i_2R_1 - i_2R_4 = 0 \tag{80}$$

$$V_{ex} - i_2(R_1 + R_4) = 0 \tag{81}$$

where

$$i_2 = \frac{V_{ex}}{R_1 + R_4} \tag{82}$$

Substituting (79) and (82) into (75) yields

$$V_{AB} + \left(\frac{V_{ex}}{R_2 + R_3}\right)(R_2) - \left(\frac{V_{ex}}{R_1 + R_4}\right)R_1 = 0 \tag{83}$$

Substituting (79) and (82) into (76) gives

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$$V_{AB} - \left(\frac{V_{\text{ex}}}{R_2 + R_3}\right)(R_3) + \left(\frac{V_{ex}}{R_1 + R_4}\right)R_4 = 0 \tag{84}$$

and

$$V_{AB} = -V_{\text{ex}}\left(\frac{R_2}{R_2 + R_3} - \frac{R_1}{R_1 + R_4}\right) = V_{\text{ex}}\left(\frac{R_1}{R_1 R_4} - \frac{R_2}{R_2 + R_3}\right) \tag{85}$$

We now deform the strain gauge (R_1 becomes $R_1 + \Delta R_1$):

$$\frac{V_{AB}}{V_{\text{ex}}} = \frac{R_1 + \Delta R_1}{(R_1 + \Delta R_1) + R_4} - \frac{R_2}{R_2 + R_3} \tag{86}$$

Equation (86) relates V_{AB} to ΔR where ΔR is related to strain.

Shuffle (86):

$$\Delta R_1 = R_1\left[\frac{\dfrac{R_4}{R_1}\left(\dfrac{V_{AB}}{V_{\text{ex}}} + \dfrac{R_2}{R_2 + R_3}\right)}{\left(1 - \dfrac{V_{AB}}{V_{\text{ex}}} - \dfrac{R_2}{R_2 + R_3}\right)} - 1\right] \tag{87}$$

Use (87) and

$$F = \frac{\Delta R/R}{\epsilon_{\text{axial}}} \tag{88}$$

This analysis [(63)–(88)] gives an example of how to design a system with strain sensors.

9 ELECTROMECHANICAL MODELING EXAMPLE

Consider the mechatronic system shown in Fig. 49. Here a permanent-magnet dc motor is used to drive a one-degree-of-freedom robotic arm in a gravity field. The motor is connected to the arm with a pair of spur gears. The arm is used to actuate a spring (which could be used to represent interaction with the environment, i.e., applying a force).

The motor can be modeled electrically as an inductor, a resistor, and a voltage source connected in parallel; see Fig. 50. If the gears are assumed to be ideal, the following assumptions may be appropriate:

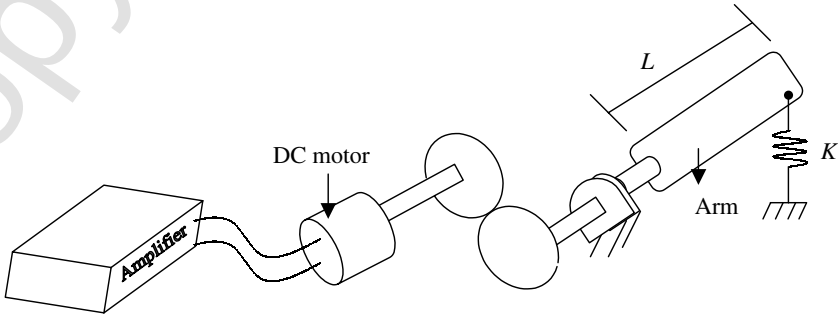


Figure 49 Example mechatronic system.

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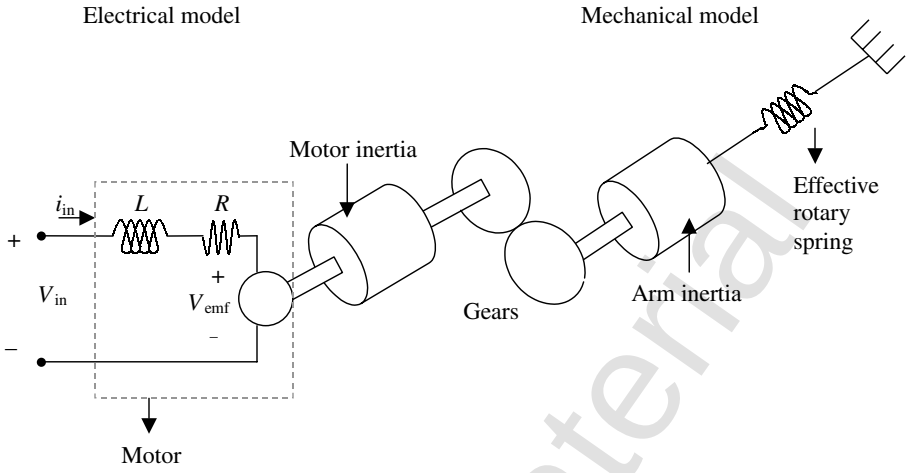


Figure 50 Model of mechatronic system.

1. No friction
2. Motor rotor and arm are the only significant masses
3. Small motions

Now the constitutive relationships for the motor and gears can be written and Kirchhoff's voltage law can be applied to the electrical circuit. Then, free-body diagrams can be created for each mechanical element; see Fig. 51:

- Motor relations:

$$\tau = K_t i_{in} = \text{torque of the motor} = (\text{torque constant})(\text{input current}) \tag{89}$$

where K_t is the torque constant.

$$V_{emf} = \omega K_e = K_e \dot{\theta}_{motor} \tag{90}$$

where K_e is the electromotive force (emf) constant.

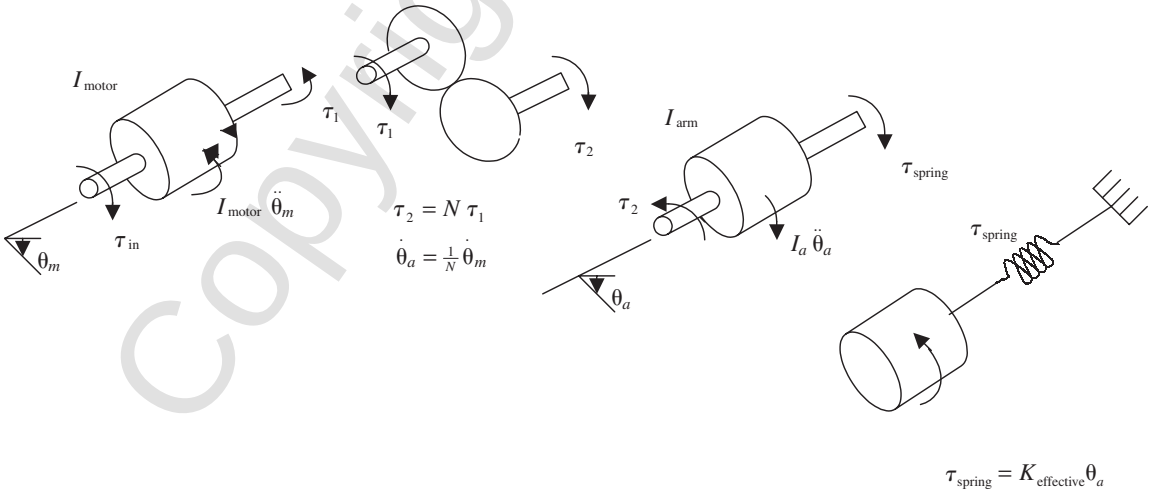


Figure 51 Free-body diagrams.

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• Electrical equations:

$$V_{\text{in}} = L\frac{di_{\text{in}}}{dt} + Ri_{\text{in}} + K_{\text{emf}}\theta_{\text{motor}} \tag{91}$$

$$\tau_{\text{in}} - \tau_1 = I_{\text{motor}}\,\ddot{\theta}_{\text{motor}} \tag{92}$$

$$\tau_2 - \tau_{\text{spring}} = I_a\ddot{\theta}_a \tag{93}$$

where $\tau_1 = (1/N)\tau_2$ and $\tau_2 = \tau_{\text{spring}} + I_a\,\ddot{\theta}_a$. Substituting into (92) yields

$$\tau_{\text{in}} - \frac{1}{N}(\tau_{\text{spring}} + I_a\ddot{\theta}_a) = I_{\text{motor}}\,\ddot{\theta}_{\text{motor}} \qquad \ddot{\theta}_a = \frac{1}{N}\,\ddot{\theta}_m \tag{94}$$

The arm can then be modeled with an effective spring constant:

$$\tau_{\text{in}} - \frac{\tau_{\text{spring}}}{N} - \frac{I_a}{N}\left(\frac{1}{N}\,\ddot{\theta}_m\right) = I_{\text{motor}}\,\ddot{\theta}_m \tag{95}$$

Then τ_{spring} can be derived as

$$\tau_{\text{spring}} = K_{\text{effective}}\theta_a \tag{96}$$

$$= LF = \underbrace{KL^2}_{K_{\text{effective}}}\,\theta_a \quad \text{(see Fig. 52)} \tag{97}$$

Substituting (95)–(97) into (94) yields

$$\tau_{\text{in}} - \frac{KL^2}{N^2}\,\theta_{\text{arm}} - \frac{I_a}{N^2}\,\ddot{\theta}_m = I_{\text{motor}}\,\ddot{\theta}_{\text{motor}} \tag{98}$$

Inertial forces — Spring force reflected to motor axis

$$\tau_{\text{in}} = \underbrace{\left(\frac{I_a}{N^2} + I_{\text{motor}}\right)}_{\text{Arm inertia reflected to motor axis}}\ddot{\theta}_m + \underbrace{\frac{KL^2}{N^2}}_{\text{Motor inertia}}\,\theta_m \tag{99}$$

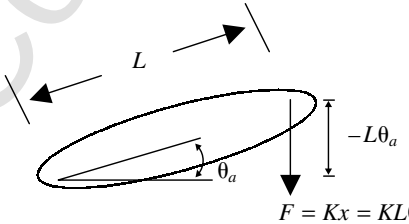


Figure 52 Free-body diagram of the arm.

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Now, since

$$\tau_{\text{in}} = K_t i_{\text{in}} \tag{100}$$

substituting (100) into (99) yields

$$K_t\, i_{\text{in}} = \left(\frac{I_a}{N^2} + I_m\right)\ddot{\theta}_m + \frac{KL^2}{N^2}\,\theta_m \tag{101}$$

Substituting (101) into (91) yields

$$V_{\text{in}} = L\,\frac{d}{dt}\left\{\frac{1}{K_t}\left[\left(\frac{I_a}{N^2} + I_m\right)\ddot{\theta}_m + \frac{KL^2}{N^2}\,\theta_m\right]\right\} + R\left\{\frac{1}{K_t}\left[\left(\frac{I_a}{N^2} + I_m\right)\ddot{\theta}_m + \frac{KL^2}{N^2}\,\ddot{\theta}_m\right]\right\} + K_e\dot{\theta}_{\text{motor}} \tag{102}$$

This differential equation relates θ_m to V_{in}. The Laplace transform can be used to obtain the transfer function:

$$\frac{\text{Output}}{\text{Input}} = \frac{\theta_m(s)}{V_{\text{in}}(s)} \tag{103}$$

Also, substituting $\theta_a = (1/N)\,\theta_m$ yields $T(s) = \theta_a(s)/V_{\text{in}}(s)$.

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